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A FUZZY-PROBABILISTIC MAINTENANCE OPTIMIZATION COST MODEL

Mariana Carvalho, Eusébio Nunes and José Telhada
Production and Systems Department
E-mail: mcarvalho@ipca.pt

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ABSTRACT

A fuzzy-probabilistic model is proposed considering that inspections and preventive maintenances are performed at periodic time intervals and the system is fully replaced less frequently when a fixed number of preventive maintenances have been completed. In order to make the model more realistic, particular emphasis will be devoted to the uncertainty of some parameters of the model, using the Fuzzy Set Theory. The main objective of the proposed model is to determine the optimal inspection and maintenance times and, simultaneously, the optimal number of inspections and preventive maintenances between replacements, minimizing the global cost of maintenance per unit of time. The applicability of the model and its performance is compared with a “crisp” model performance.

INTRODUCTION

The Fuzzy Set Theory has been extensively studied in the past 30 years. It was largely motivated by the need for a more expressive mathematical structure to deal with human factors and it has a major impact on industrial engineering, including on maintenance planning. In fact, this is an area where large amounts of data are quickly processed and where almost exists total dependence of historical references and of the quality and experience of experts and maintenance engineers. Therefore, the Fuzzy Set Theory has been playing a role of particular relevance with regard to delineating maintenance actions, providing critical support in specific areas, such as, for instance, the detection of imminent failures.
During the last decade, several models in maintenance planning have been incorporating uncertainty of their parameters (e.g. the mean time between failures, the mean time to repair or the probability of non detecting a failure) by using fuzzy numbers. Some of these studies can be found in Hong (2006), Khanlari *et al* (2008). Al-Najjar and Alsyouf (2003), and Lu and Sy (2009) developed models that support decision making in choosing the most efficient maintenance technique. Nevertheless, most of the current literature on maintenance modeling simply omits the uncertainty that is inherent to real data and parameters.
This paper develops a fuzzy model for periodic imperfect preventive maintenance policies. It is considered that the system failures can be grouped into two categories or modes: maintainable or non-maintainable, analogously to Castro (2009). However, the model presented herein assumes that failures can be immediately revealed (with probability *p*) or only revealed in the next inspection/preventive maintenance (with probability 1-*p*). Moreover, in the present work, maintenance actions are not instantaneous. As it is well known, from the reliability literature, maintenance time is frequently ignored during the optimization of maintenance policies. This fact may lead to unrealistic results. In this study, the uncertainty of costs and reliability parameters is not omitted by the model, being represented through fuzzy numbers.

Notation

T	Inspection period
τ	Expected time of unavailability due to preventive maintenance actions (at inspections) and replacement.
N	Number of time intervals (of length T) between two successive full replacements of the system.
c_p	Cost of inspection plus preventive maintenance.
c_{d1}	Cost of not detecting a bad functioning for maintainable failures.
c_{d2}	Cost of not detecting a bad functioning for non-maintainable failures.
c_r	Cost of replacement of the system.
c_{m1}	Cost of minimal repair for maintainable failures.
c_{m2}	Cost of minimal repair for non-maintainable failures.
c_u	Cost of system unavailability.
$H_1(t)$	Cumulative maintainable failure rate at time t..
$H_2(t)$	Cumulative non-maintainable failure rate at time t.
p	Probability by which a failure is (immediately) detected when it occurs.
a	Adjustment factor. If $a>1$ it represents the effect of the wear-out of the system (due to non-maintainable failures) in the occurrence of maintainable failures.
$C(T, N)$	Expected maintenance cost per unit of time as a function of T and N.

MAINTENANCE MODEL

Whenever a failure is detected, a minimal repair is performed. A minimal repair action returns the system to the condition as bad as it was immediately before the occurrence of the failure. A cycle is defined as the time interval between two consecutive renewals of the system, that is, $N(T+\tau)$.

At times $kT+(k-1)\tau$ (k=1, 2, …, N-1) of a given cycle, the system is inspected and preventively maintained. The inspection is performed in order to detect unrevealed failures, which occur with probability 1-p. After the inspection, a preventive maintenance takes place immediately. Preventive maintenance is considered imperfect since that it can only reduce the failure rate of the maintainable failures.

Removing degradation due to non-maintainable failures is only possible by making a complete overhaul, which restores he system to as good as new condition. At time $NT+(N-1)\tau$, N= 1, 2, …, the system is replaced.

It is recognized the existence of costs due to inspections and preventive maintenances, repairs, replacements, non detection of failures and unavailability.

Our aim is to determine the time interval between any two successive preventive maintenances and the number of preventive maintenances between any two successive replacements that must be performed in order to minimize the global cost of maintenance per unit of time (Carvalho *et al*, 2009):

$$C(T,N) = \frac{c_1 H_1(T)\sum_{k=0}^{N-1} e^{(a-1)H_2(kT)} + c_2 H_2(NT) + c_r + (N-1)c_p + Nc_u}{N(T+\tau)}$$

where $c_1=c_{m1}+(1-p)c_{d1}$ and $c_2=c_{m2}+(1-p)c_{d2}$.

It is important to note that almost all the parameters of this cost model are very difficult to estimate accurately. For example, it is well-known that the unavailable time and the adjustment factor are uncertainty. Likewise, the probability by which a failure is detected when it occurs, as well as the not detecting costs, are parameters very difficult to obtain or estimate. Therefore, it is assumed these parameters as fuzzy numbers, so the cost model becomes:

$$\tilde{C}(T,N) = \frac{\tilde{c}_1 H_1(T)\sum_{k=0}^{N-1} e^{(\tilde{a}-1)H_2(kT)} + \tilde{c}_2 H_2(NT) + c_r + (N-1)c_p + Nc_u}{N(T+\tilde{\tau})}$$

The fuzziness nature of the parameters is propagated to the results, giving rise to more realistic solutions.

CONCLUSIONS

The problems of finding the optimal length between successive preventive maintenances and the optimal number of preventive maintenances between replacements have been studied. In order to better model uncertainty in some of the parameters, it was considered the Fuzzy Set Theory.

The applicability of the model and its performance has been compared with an equivalent rigid-parameter model. The former seems to give rise to more realistic solutions. In complex systems it is impossible to have a perfect knowledge about the involved parameters (failure rates, unavailability times, …) and about their interdependency relationships. Considering these results as "crisp" numbers is assuming that there are no uncertainty in these data. In fact, the uncertainty of data is intrinsic and it is not probabilistic. To overcome these limitations, the application of the Fuzzy Set Theory seems to be an interesting approach.

Further developments to the model herein proposed may consider algorithmic tools that support the methodologies theoretically developed.

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AUTHOR BIOGRAPHIES

MARIANA CARVALHO holds a degree and a Masters in Mathematics. Since 2006, she is studing for a PhD. in Industrial Management and Systems. She is Assistant Professor in School of Technology of the Institut Polytechnic of Cávado and Ave. Her research interests include reliability and maintenance of systems, fuzzy modeling and performance evaluation of manufacturing systems.
Her e-mail address is : mcarvalho@ipca.pt.

EUSÉBIO NUNES holds a degree in production engineering from the School of Engineering of the University of Minho, Portugal (since 1992). He received his Master in IST-Lisbon in 1996 and a Ph.D. from Faculty of Engineering of the University of Porto (FEUP), Portugal. He is Assistant Professor at University of Minho, currently teaching industrial engineering. His research interests include reliability of systems, fuzzy modeling and performance evaluation of manufacturing systems.

JOSÉ TELHADA holds a Master degree in the area of transport planning and engineering from the Institute for Transport Studies of the University of Leeds, since 1993. He received a Ph.D. from the same institution in 2001. He is Assistant Professor at University of Minho, currently teaching industrial engineering. His research interests include transportation planning, logistics and reliability of systems.